lim sup and lim inf in three languages

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$$\omega \in \liminf A_n \iff \exists m_0 \in \mathbb{N} : \omega \in \bigcap_{n \geqslant m_0} A_n$$

$$\iff \exists m_0 \in \mathbb{N} \, \forall n \geqslant m_0 : \omega \in A_n$$

$$\iff \omega \text{ liegt in fast allen } A_n \ (n \in \mathbb{N}) \tag{DE}$$

$$\iff \omega \in A_n \text{ für schließlich alle } n \in \mathbb{N}$$

$$\iff \omega \in A_n \text{ für fast alle } n \in \mathbb{N}$$

$$\iff \omega \in A_n \text{ taken from (countably) infinitely many } n \in \mathbb{N} \tag{EN}$$

$$\iff \omega \in A_n \text{ for all except finitely many } n \in \mathbb{N} \text{ (i.e., for cofinitely many } n)$$

$$\iff \omega \text{ in ultimately all of } A_n \ (n \in \mathbb{N})$$

$$\iff A_n \text{ ultimately (ult.)}$$

$$\iff \omega \text{ appartient à tous les } A_n \text{ à partir d'un certain rang} \tag{FR}$$

$$\iff \omega \in A_n \left[\mathbb{1}_{A_n}(\omega) = 1 \right] \text{ pour } n \in \mathbb{N} \text{ suffisament grand}$$

and

$$\omega \in \limsup A_n \iff \forall m \in \mathbb{N} : \omega \in \bigcup_{n \geqslant m} A_n$$

$$\iff \forall m \in \mathbb{N} \, \exists n_0(m) \geqslant m : \omega \in A_{n_0(m)}$$

$$\iff \omega \text{ ist in unendlichen vielen der } A_n \text{ enthalten } (n \in \mathbb{N}) \tag{DE}$$

$$\iff \omega \in A_n \text{ für unendlich viele } n \in \mathbb{N}$$

$$\iff A_n \text{ unendlich oft (u.o.)}$$

$$\iff \omega \text{ appears in infinitely many of the } A_n \tag{EN}$$

$$\iff \omega \in A_n \text{ for all but finitely many } n \in \mathbb{N} \text{ (i.e., cofinitely many } n)$$

$$\iff A_n \text{ for infinitely many } n \in \mathbb{N}$$

$$\iff A_n \text{ infinitely often (i.o.)}$$

$$\iff \omega \text{ appartient à } A_n \text{ pour une infinité d'indices } n \tag{FR}$$

$$\iff \omega \in A_n \left[\mathbb{1}_{A_n}(\omega) = 1 \right] \text{ pour une infinité de } n \in \mathbb{N}$$

$$\iff \omega \in A_n \text{ infiniment souvent (i.s.)}$$

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